AN OCEAN IN YOUR COMPUTER

3.1 INTRODUCTION TO OCEAN MODELLING: IT'S ALL IN THE NUMBERS

VIDEO DURATION– 05:38

In this lecture we are going to learn about ocean models, how they are constructed and what they can usefully tell us about the real oceans. This lecture was written by Dr Andrew Coward, a researcher at the National Oceanography Centre in the UK.

First though, it is worth reviewing the general concept of a mathematical model and the role of simulation in modern scientific research.

A **mathematical model** is a description of a system using mathematical concepts and language.

This is probably much more familiar to you than you realise.

Take, for example, **Newton's Second Law of Motion** that he originally stated as:

The change of momentum of a body is proportional to the impulse applied to that body, and happens along the straight line on which the impulse is applied.

This is more recognisable for objects of constant mass when it reduces to Force equals Mass times Acceleration.

Or "using mathematical concepts and language" as the classic equation **F = ma**

So what have we gained by expressing a physical law in mathematical terms?

For starters, the equation is more concise and unambiguous, but the importance of a mathematical model is that it can help to explain a system, be used to study the effects of different components, and be used to make predictions about behaviour.

In modern research there are many examples where testing systems and hypotheses in the real world are simply impossible, socially or environmentally unacceptable, prohibitively expensive or too time consuming.

Many issues in oceanographic research fall in one or more of these categories.

So how can a mathematical model help to make predictions? Take Newton's infamous apple, for example. Newton would have been aware of Galileo's observations that objects fall to Earth with a constant acceleration, '**g**'.

But acceleration is just the rate of change of velocity:

$$dv/dt = g$$

Which we can solve, if we know an initial velocity of V_0:

$$V = gt + V_0$$

But velocity is just the rate of change of displacement so, assuming an initial displacement of X_0 we can get the following:

$$dx/dt = V = gt + V_0$$

$$x = \tfrac{1}{2} gt^2 + V_0 t + X_0$$

Hence, assuming the apple was initially at rest and its displacement is measured from the point of release at height H metres above Newton's head, Newton could have predicted that he had precisely $\sqrt{(2H/g)}$ seconds to dodge the impact.

Fast mover or not, there is little evidence that the apple ever actually hit Newton.

We seem to be a long way from an ocean model but the same principles apply. Things get more complicated when the body to which forces are applied:

- can change shape, phase and composition
- is subject to internal stresses and strains
- has a host of external forces acting on it, and
- can move over distances great enough that the rotation of the Earth cannot be ignored.

Nevertheless, thanks to many great scientists - both past and present - we can formulate a series of equations that describe the forces acting on a parcel of ocean water, and hence how it moves and changes subject to those forces.

There are plenty of caveats though:

- The oceanic system is vast with spatial motions on the scale of 100's or 1000's of kilometres down to molecular diffusion, and changes happening from instantaneous to over century-long time-scales. The chances of capturing every process affecting a water parcel in our mathematical description are very small.

- The equations describe how the parcel evolves and moves with time, but we cannot know accurately the initial state of the ocean on which to base our predictions. Again, it is just too big and sparsely observed.

- The external forces acting on the ocean are not known precisely. This is true not only for the rapidly changing atmospheric conditions at the surface, but also for other inputs, such as buoyancy forces due to geothermal heating on the seabed or freshwater input from rivers, glaciers and sea-ice.

All these lead to levels of uncertainty in any results from our ocean models, but there is an even more fundamental issue to be addressed first.

The set of equations describing a realistic ocean cannot be solved using traditional mathematical techniques. So what good is a set of equations that describes ocean properties for every point and every time, if it is impossible to solve those equations?

The answer is that we can compromise, and accept that it is sufficient to know the ocean state at only some places and at only some times.

If we do this, and we make some assumptions about how the ocean varies in between these discrete places and times, then we can derive approximate solutions for each discrete place and time. By this process of discretisation we have turned our ocean model into a numerical ocean model and will, almost certainly, need to employ computers to handle the large number of calculations required to derive values for each location and time.

This then is the essence of numerical ocean modelling and, indeed, is the same process used to derive Numerical Weather Prediction programs that provide our daily weather forecasts.

The choices we make about how many discrete places and how they are spread throughout our volume of interest, will determine the type of ocean model and the uses that it can be put to.

In summary this lecture has:

- Introduced the concept of mathematical modelling,
- Explained how models can be used to make predictions,
- Described some of the sources of uncertainty in those predictions for ocean models, and
- Discussed how to formulate discreet versions of the models that we can solve using computers.